Project Name: Graph Analysis via Density of States

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Researchers from many disciplines study spectra to understand the structure and composition of mathematical objects and physical systems. For large systems, complete spectral information costs too much, whether it is gathered computationally or measured experimen- tally; hence, most spectral analysis methods use partial information about the eigenvalues and eigenvectors of some matrix or operator. Broadly speaking, these methods use either

- a few eigenvalues (usually the largest or smallest) and associated eigenvectors;
- invariants that are simple functions of all the eigenvalues; or
- densities of eigenvalues (aka the density of states), possibly weighted by how important they are to a particular node (i.e. a local density of states). All three approaches are used in spectral geometry, spectral graph theory, and in applications to physics and engineering systems. But in the study of complex networks, information about spectral densities is rarely used. Our main research objective is to apply to network science the local and global "den- sity of states" spectral analysis techniques common in other areas. Our approach is cross- disciplinary, bringing to bear computational methods and analytical tools from condensed matter physics and spectral geometry. Though we build on ideas from other disciplines, taking full advantage of these techniques in the context of network science will involve novel computational tools and mathematical analysis. The intellectual merit of the pro- posal lies in both these new methods of computation and analysis and in the demonstration of how these methods can be used to classify and analyze complex networks. The broader impact of our research will be felt by network science researchers who use the software we produce, and by graduate and undergraduate researchers exposed to interdisciplinary research through their involvement in the project. Our proposed work includes three main activities:

1. Extending methods known in the condensed matter physics literature, we will develop novel high-performance methods to compute local and global spectral den- sities (densities of state) for "natural" large-scale networks.

2. We will develop novel graph decomposition techniques that use spectral densi- ties information to infer to what extent a graph can reasonably be decomposed into subgraphs

that follow well-understood motifs. We will use localized spectral densities as "fingerprints" for understanding the roles of each node in these motifs and in the network as a whole.